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# An air-ice feedback mechanism for the migration of the marginal ice zone

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## An Ice/Air Feedback Mechanism for the Migration of the Marginal Ice Zone

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The aim of this study is to investigate, by means of a coupled ice/air model, the generation of strong along-ice-edge winds in the MIZ, the formation of the ice edge jet, and the instability criteria of ice drift. Ice-drift observations in the Greenland Sea from 28 April to 3 September 1978 (Figure 1) show two different types of ice motion. Oscillation occurs mostly in spring (April, May, and June) and nonoscillation takes place in summer (July and August). Ice melts and becomes thinner from winter to summer. Although ocean eddies probably produce oscillations in drift track, another relation between the pattern of ice motion and ice thickness may still exist. One purpose of the present work is to find this relation.

A possible mechanism for the air/ice interaction is presented in Figure 2. The low-level air flow generated by differential surface heating is waterward (icebreeze) and equatorward along the ice edge in both eastern Greenland and the eastern Antarctic Peninsula and influences the ice through surface air stress. However, movement of the ice edge in the MIZ toward warm water changes the thermal conditions near the surface and produces an air surface temperature gradient across the ice edge.

The ice/air interaction model depicted in this article is intended to simulate the main physical processes and to determine some instability criteria for the prediction of MIZ migration.

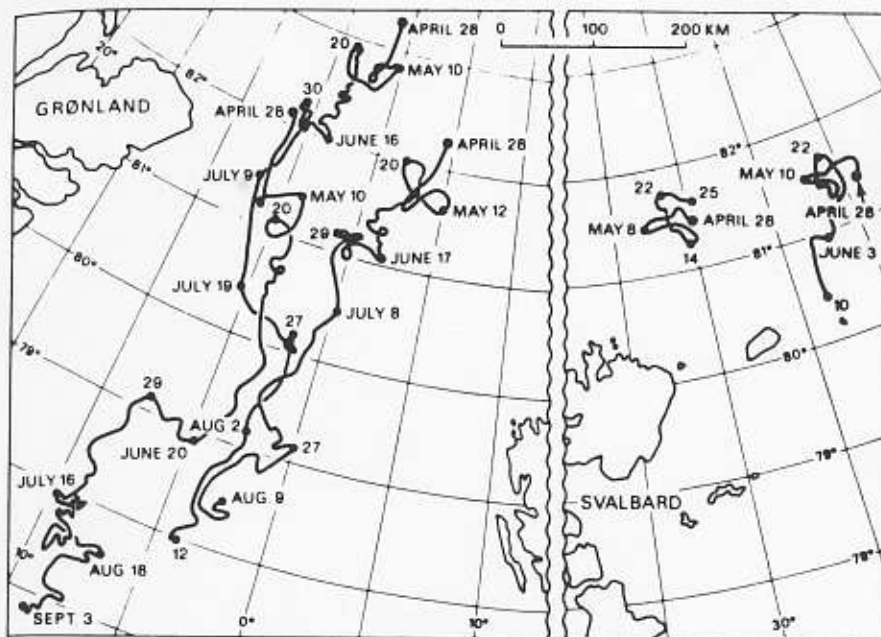


Fig. 1. Ice-drift observations in the Greenland Sea.

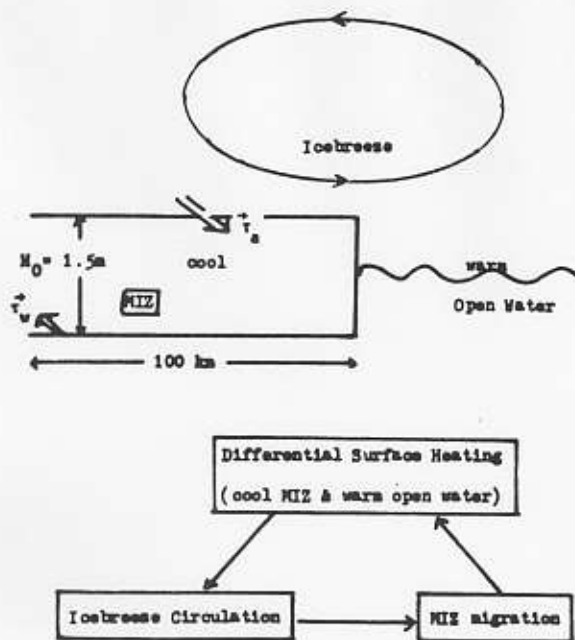


Fig. 2. Ice/air/water coupled system.

#### Thermally forced boundary layer air flow

The surface isotherms, which are nearly parallel to the eastern coast of Greenland over the Greenland Sea (Prik, 1959), indicate that air temperature monotonically increases waterward near the MIZ. Such differential surface heating generates a local air flow near the MIZ. In this section we utilize a planetary boundary layer model treated by Kuo (1973) and Chu (1985) to simulate a thermally forced boundary layer air flow. The coordinate system is chosen as moving with the edge of the MIZ. The  $x$ -axis is in the cross-edge direction, and the  $y$ -axis parallels the ice edge, as shown in Figure 3. The  $x$ -coordinate's unit length is twice the MIZ width,  $L$  (200 km), and that of the vertical coordinate is  $\Omega = (\nu/\Omega)^{1/2}$ , where  $\nu$  is the vertical eddy viscosity and  $\Omega$  is the angular velocity of the earth's rotation. The line  $x = 1/2$  is located at the ice edge. The MIZ covers the zone ( $0 < x < 0.5$ ,  $y$ ) where the  $y$ -axis ( $x = 0$ ) is near the boundary between the MIZ and the interior ice pack. It is considered that spatial variations in the MIZ are much larger perpendicular to the ice edge than parallel to it, so derivatives with respect to  $y$  are assumed to be zero. Since acceleration of the ice edge is small compared to that of the air flow, we may ignore the inertial force due to the use of a coordinate system (moving relative to the earth).

The potential temperature of air is divided into two parts: a basic state  $\theta_{B0}(z)$  and perturbation  $\theta'_s$ . The basic state is given by

$$\theta_{B0}(z) = \theta_{B0} + (N^2 \theta_{B0} \delta / g) z, \quad (1)$$

where  $\theta_{B0}$  is the basic air potential temperature at the surface and  $N$  is the Brunt-Väisälä frequency.

Waterward migration of the MIZ increases the surface temperature gradient, and iceward migration of the MIZ decreases the surface temperature gradient. Surface temperature perturbation is parameterized as

$$\theta'_s = -DT_0[1 + \zeta(t)] \cos \pi x, \quad (2)$$

where  $DT_0$  is the mean surface temperature difference across the MIZ and  $\zeta(t)$  is the nondimensional displacement of the ice edge. The coordinates and atmospheric variables are nondimensionalized by setting

$$(x, z, t) = (xL, z\delta, tT),$$

$$s' = \theta'_s / \theta_{B0} = (DT_0 / \theta_{B0}) s \quad (3)$$

$$(u, v, w) = U(u, v, w\delta/L),$$

$$P_s = \rho_a(g\delta DT_0 / \theta_{B0}) P,$$

where

$$U \equiv g\delta DT_0 / (2L\Omega\theta_{B0}) \quad (4)$$

is the scale of the icebreeze wind and  $T$  is the time scale for the change of surface heating due to the movement of the ice edge. If we assume that the local air flow satisfies the modified Boussinesq approximation (Kuo, 1973), then the vorticity equation, the momentum equation (both in the  $y$  direction), and the heat equation for air disturbances

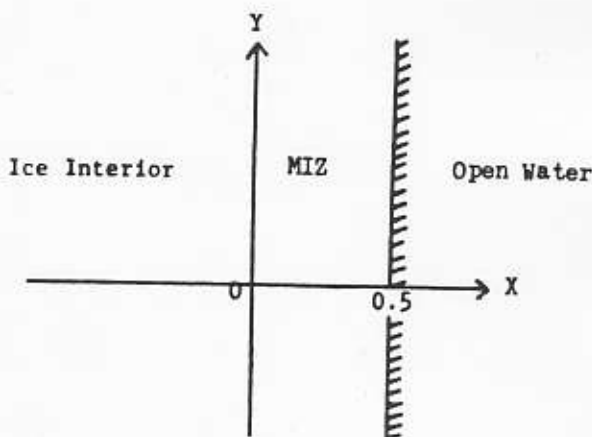


Fig. 3. The model MIZ and the coordinate system.

generated by differential heating near the MIZ are written by (Chu, 1985)

$$E\nabla_z^2 \nabla_z^2 = f_0 \partial v / \partial z - \partial s / \partial x, \quad (5)$$

$$E\nabla_z^2 v = -f_0 \partial \psi / \partial z, \quad (6)$$

$$E\nabla_z^2 s = R_1 \partial \psi / \partial x, \quad (7)$$

and

$$\begin{aligned} u &= -\partial \psi / \partial z, \quad w = \partial \psi / \partial x, \\ \nabla_z^2 &\equiv \delta^2 \nu_h / (L^2 \nu) \partial^2 / \partial x^2 + \partial^2 / \partial z^2, \\ \nabla^2 &\equiv \delta^2 / L^2 \partial^2 / \partial x^2 + \partial^2 / \partial z^2 \equiv \partial^2 / \partial z^2, \end{aligned} \quad (8)$$

where  $f_0 = \sin \lambda$ ,  $\lambda$  is the latitude, and

$$E = \nu / (2\Omega \delta^2) = 1/2, \quad R_1 \equiv \delta^2 N^2 / (4L^2 \Omega^2) \quad (9)$$

are the Ekman number and global Richardson number, respectively. Eliminating  $v$  and  $s$  from Equations 5 through 7, we find that the streamfunction satisfies the following partial differential equation:

$$(\frac{1}{4} \nabla_z^4 + f_0^2) \partial^2 \psi / \partial z^2 + R_1 \partial^2 \psi / \partial x^2 = 0. \quad (10)$$

We solve Equation 10 for streamfunction  $\psi$ , and obtain the solutions of  $v$  and  $s$  from Equations 6 and 7 after substituting  $\psi$ . The local air flow is thermally forced by differential surface heating as indicated in Equation 2; therefore, the streamfunction should be written as

$$\psi(x, z, t) = \bar{\psi}(z, t) \sin \pi x. \quad (11)$$

The boundary conditions in the vertical direction are derived as follows. The dependent variables should remain finite as  $z \rightarrow \infty$ , i.e.,

$$\lim_{z \rightarrow \infty} (|\psi|, |\partial \psi / \partial z|, |v|, |s|) < \infty. \quad (12)$$

The boundary conditions at  $z = 0$  are:

$$\begin{aligned} \psi &= 0, & \partial \psi / \partial z &= K \partial^2 \psi / \partial z^2, \\ v &= K \partial v / \partial z, & s &= -[1 + \zeta(t)] \cos \pi x, \end{aligned} \quad (13)$$

where  $K$  is a measure of the effective depth of the constant stress-sublayer (Kuo, 1971). Substituting Equation 11 into Equation 10 we obtain the following sixth-order ordinary differential equation for the Fourier coefficient  $\bar{\psi}$ :

$$\begin{aligned} d^6 \bar{\psi} / dz^6 - 4\gamma d^4 \bar{\psi} / dz^4 + 4(f_0^2 + \gamma^2) \\ d^2 \bar{\psi} / dz^2 - 4\pi^2 R_1 \bar{\psi} &= 0 \end{aligned} \quad (14)$$

where

$$\gamma \equiv \pi^2 \delta^2 \nu_h / (2L^2 \nu). \quad (15)$$

The general solution of Equation 14 has the following form:

$$\bar{\psi}(z, t) = [1 + \zeta(t)] \sum_{j=1}^6 \bar{a}_j e^{\lambda_j z}, \quad (16)$$

where the eigenvalues  $\lambda_j$  ( $j = 1, 2, \dots, 6$ ) are the roots of the succeeding sixth-order algebraic equation:

$$\lambda^6 - 4\gamma \lambda^4 + 4(f_0^2 + \gamma^2) \lambda^2 - 4\pi^2 R_1 = 0. \quad (17)$$

Table 1 lists all the eigenvalues under typical stratification at three distinct latitudes ( $65^\circ$ ,  $70^\circ$ , and  $75^\circ$ ). Notice that  $f_0^2$  appears in Equation 17, so the eigenvalues in Table 1 represent both northern and

**Table 1. The eigenvalues of the boundary layer air flow model at three different latitudes.**

Latitude		
65°	70°	75°
(-0.32371, 0)	(-0.31223, 0)	(-0.30378, 0)
(-1.02800, 0.87796)	(-1.04508, 0.89586)	(-1.05823, 0.90973)
(-1.02800, -0.87796)	(-1.04508, -0.89586)	(-1.05823, -0.90973)
(0.32371, 0)	(0.31223, 0)	(0.30378, 0)
(1.02800, 0.87796)	(1.04508, 0.89586)	(1.05823, 0.90973)
(1.02800, -0.87796)	(1.04508, -0.89586)	(1.05823, -0.90973)

southern polar regions. According to the upper boundary conditions listed in Equation 12 we must set up coefficients that correspond to those eigenvalues with positive real parts to zero. Consequently the general solution Equation 16, satisfying the top boundary conditions, may be written by:

$$\bar{\psi}(z, t) = [1 + \zeta(t)] \sum_{j=1}^3 \bar{a}_j e^{\lambda_j z}, \quad (18)$$

where the eigenvalues  $\lambda_j$  all have the negative real parts. Substituting Equation 18 into 11 we obtain the streamfunction

$$\psi(x, z, t) = \sum_{j=1}^3 \bar{a}_j [1 + \zeta(t)] e^{\lambda_j z} \sin \pi x. \quad (19)$$

Integrating the momentum equation (6) and heat equation (7) with respect to  $z$  after substituting Equation 19, we find that  $v$  and  $s$  are given by

$$v(x, z, t) = [\bar{b} e^{-\sqrt{2\gamma}z} - f_0] \sum_{j=1}^3 \bar{a}_j \lambda_j F(\lambda_j, \gamma) e^{\lambda_j z} [1 + \zeta(t)] \sin \pi x, \quad (20)$$

$$s(x, z, t) = [\bar{c} e^{-\sqrt{2\gamma}z} + \pi R_1] \sum_{j=1}^3 \bar{a}_j F(\lambda_j, \gamma) e^{\lambda_j z} [1 + \zeta(t)] \cos \pi x. \quad (21)$$

where

$$F(\lambda, \gamma) \equiv 1/(\lambda^2/2 - \gamma). \quad (22)$$

Substituting solutions 19-21 into the vorticity equation (5) we find that

$$\bar{c} = \sqrt{2\gamma} f_0 \bar{b} / \pi. \quad (23)$$

Substituting solutions 19-21 into the surface boundary conditions listed in Equation 13, we obtain the following four algebraic equations for  $\bar{a}_j$  and  $\bar{b}$ .

$$\sum_{j=1}^3 \bar{a}_j = 0 \quad (24)$$

$$\sum_{j=1}^3 \lambda_j (1 - K \lambda_j) \bar{a}_j = 0 \quad (25)$$

$$-f_0 \sum_{j=1}^3 \lambda_j (1 - K \lambda_j) F(\lambda_j, \gamma) \bar{a}_j + \bar{b} (1 + K \sqrt{2\gamma}) = 0 \quad (26)$$

$$\pi R_1 \sum_{j=1}^3 F(\lambda_j, \gamma) \bar{a}_j + \sqrt{2\gamma} f_0 \bar{b} / \pi = -1. \quad (27)$$

The four constants  $\bar{a}_1$ ,  $\bar{a}_2$ ,  $\bar{a}_3$ , and  $\bar{b}$  can be readily obtained by solving the four linear nonhomogeneous algebraic equations.

Figures 4-6 reveal the solutions for zero ice motion at  $\lambda = 75^\circ$ ,  $DT_0 = 6^\circ\text{C}$ , and  $N = 10^{-2} \text{ s}^{-1}$ . Figure 4 shows the distribution of air temperature

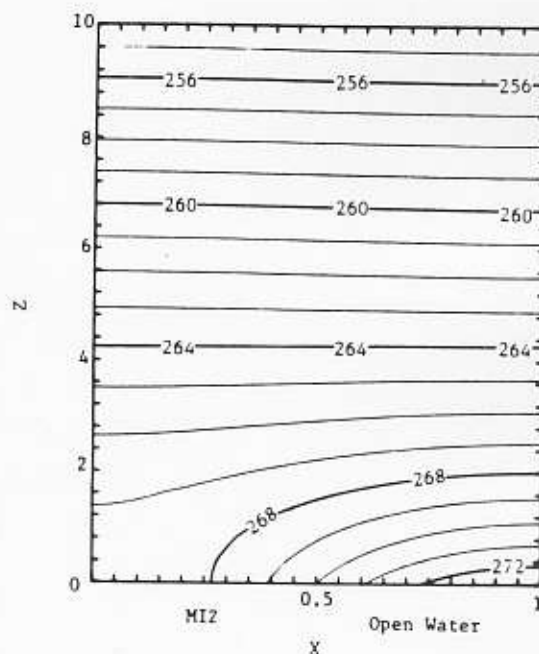


Fig. 4. The distribution of air temperature ( $^{\circ}\text{K}$ ) in the cross-ice section.



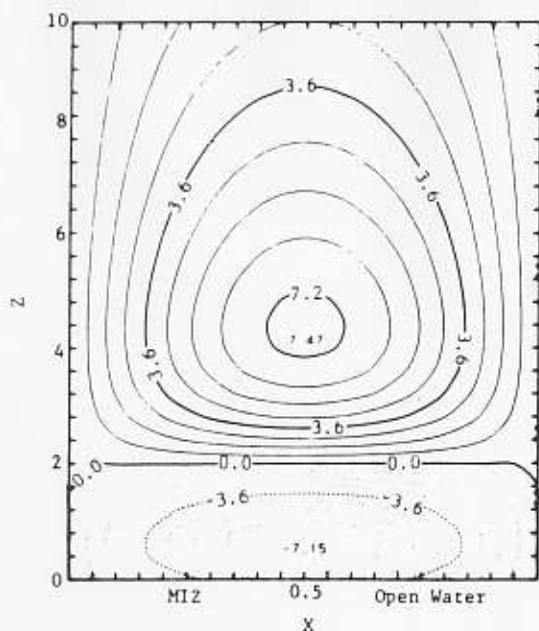


Fig. 5. The distribution of thermally forced along-ice-edge winds ( $\text{m s}^{-1}$ ) in the cross-ice section.

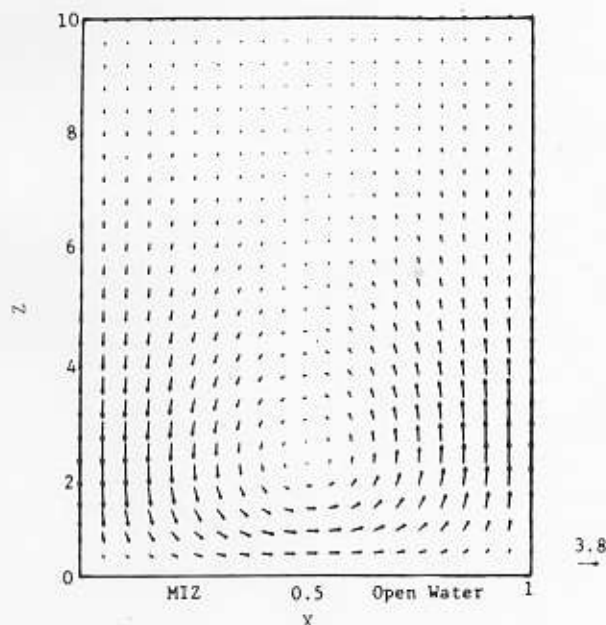


Fig. 6. Thermally forced icebreeze circulation in the cross-ice section with  $u$  in  $\text{m s}^{-1}$  and  $w$  in  $\text{cm s}^{-1}$ .

in the cross-ice section. The horizontal temperature gradient is very deep near the ice edge and decreases with altitude. Above  $z = 3$  ( $z_e = 800 \text{ m}$ ) there is almost no horizontal temperature gradient. Figure 5 shows the distribution of the wind along ice-edge  $v$  in the cross-ice section. The long-shore wind is equatorward at low levels (below  $z = 2$ ) and poleward at high levels. This strong equatorward wind along the ice edge near the MIZ (around  $7 \text{ m s}^{-1}$ ) is consistent with the observations along the eastern Antarctic Peninsula (Schwerdtfeger, 1979). Figure 6 shows the cross-ice circulation. The maximum value of the ice breeze, which is around  $3 \text{ m s}^{-1}$ , is located at the ice edge.

#### Ice drift model

Suppose that the MIZ is considered to be pack ice with linear viscous rheology and constant shear and bulk viscosities. The MIZ has a mean thickness of  $H_0$ . The linearized momentum equation and continuity equation are written by:

$$\begin{aligned} \partial u_i / \partial t_* &= (\bar{\xi} + \bar{\eta}) \partial^2 u_i / \partial x^2 \\ &+ f v_i + (\tau_{ax} + \tau_{wx}) / (\rho_i H_0) - g \partial h_i / \partial x, \end{aligned} \quad (28)$$

$$\begin{aligned} \partial v_i / \partial t_* &= \bar{\eta} \partial^2 v_i / \partial x^2 - f u_i \\ &+ (\tau_{ay} + \tau_{wy}) / (\rho_i H_0) \end{aligned} \quad (29)$$

$$\partial h_i / \partial t_* = -(H_0 / L) \partial u_i / \partial x, \quad (30)$$

where  $f (= 2\Omega \sin \lambda)$  = Coriolis parameter

$\rho_i$  = ice density

$u_i, v_i = \mathbf{V}_i$ , ice velocity

$\tau_{ax}, \tau_{ay} = \tau_a$ , wind stress

$\tau_{wx}, \tau_{wy} = \tau_w$ , water stress

$h_i$  = ice thickness perturbation generated by forcing terms

$x$  = nondimensional horizontal coordinate

and  $\bar{\eta}$  and  $\bar{\xi}$  are defined by

$$\bar{\xi} \equiv \xi / (\rho_i H_0 L^2), \quad \bar{\eta} \equiv \eta / (\rho_i H_0 L^2), \quad (31)$$

where  $\eta$  and  $\xi$  are the shear and bulk viscosities of ice. The air and water stresses are given by

$$\tau_a = C'_a \mathbf{V}_a, \quad (32)$$

$$\tau_w = C'_w (\mathbf{V}_w - \mathbf{V}_i), \quad (33)$$

where  $V_a$  is the surface air velocity,  $V_w$  the surface water velocity, and  $C'_a$  and  $C'_w$  the dimensional air and water drag coefficients.  $C'_a$  is further given by

$$C'_a = C_a \rho_a U [(\partial \psi_0 / \partial z)^2 + v_0^2]^{1/2} \big|_{z=0, x=0.5}, \quad (34)$$

where  $\psi_0$  and  $v_0$  are obtained by Equations 19 and 20. In contrast to the treatments by Hibler (1984), Lepparanta and Hibler (1984), and Lepparanta (1984), the surface air and water stresses  $\tau_a$  and  $\tau_w$  are directly computed by the boundary layer flow model presented in the previous section. Substituting the expressions for air and water stresses (32) and (33) into Equations 28 and 29 and assuming that  $V_w = 0$ , we obtain

$$\partial u_i / \partial t_* = (\bar{\xi} + \bar{\eta}) \partial^2 u_i / \partial x^2 - \bar{C}_w u_i + f v_i$$

$$\bar{C}_a u_{a0} [1 + \zeta(t)] \sin \pi x - g/L \partial h_i / \partial x, \quad (35)$$

$$\partial v_i / \partial t_* = \bar{\eta} \partial^2 v_i / \partial x^2 - \bar{C}_w v_i - f u_i$$

$$+ \bar{C}_a v_{a0} [1 + \zeta(t)] \sin \pi x, \quad (36)$$

where

$$\bar{C}_a \equiv C'_a / (\rho_i H_0), \quad \bar{C}_w \equiv C'_w / (\rho_i H_0),$$

$$u_{a0} = -g \delta DT_0 / (2L \Omega \theta_{B0}) \sum_{j=1}^3 \bar{a}_j \lambda_j,$$

$$(37)$$

$$v_{a0} = g \delta DT_0 / (2L \Omega \theta_{B0}) [\bar{b} - f_0 \sum_{j=1}^3 \bar{a}_j \lambda_j F(\lambda_j, \gamma)].$$

Equations 30, 35, and 36 are the basic equations for ice motion. The solutions of these two equations should have the same Fourier component as the forcing terms, i.e.,

$$u_i(x, t_*) = \bar{u}_i(t_*) \sin \pi x,$$

$$v_i(x, t_*) = \bar{v}_i(t_*) \sin \pi x. \quad (38)$$

By definition,  $\zeta(t)$  is the nondimensional ice-edge displacement in the  $x$ -direction. It is related to ice velocity by

$$L d\zeta / dt_* = u_i(1/2, t_*) = \bar{u}_i(t_*). \quad (39)$$

Eliminating two of the three dependent variables,  $u_i$ ,  $v_i$ , and  $h_i$ , from Equations 30, 35, and 36 we get

the following third-order equation for the coefficients:

$$[D^3 + (\alpha_1 + \alpha_2)D^2 + (\pi^2 g H_0 / L^2 - \bar{C}_a u_{a0} / L$$

$$+ \alpha_1 \alpha_2 + f^2)D + \alpha_2(\pi^2 g H_0 / L^2 - \bar{C}_a u_{a0} / L)$$

$$- f \bar{C}_a v_{a0} / L](\bar{u}_i, \bar{v}_i) = 0, \quad (40)$$

where

$$D = d/dt_*,$$

$$\alpha_1 \equiv \pi^2(\bar{\xi} + \bar{\eta}) + \bar{C}_w,$$

$$\alpha_2 \equiv \pi^2 \bar{\eta}^2 \bar{C}_w \quad (41)$$

The general solutions of Equation 40 have the following form:

$$\bar{u}_i(t_*) = \sum_{j=1}^3 d_j \exp(\mu_j t_*),$$

$$(42)$$

$$\bar{v}_i(t_*) = \sum_{j=1}^3 e_j \exp(\mu_j t_*).$$

The eigenvalues  $\mu_1, \mu_2, \mu_3$  are the roots of the following equation:

$$\mu^3 + (\alpha_1 + \alpha_2)\mu^2$$

$$+ (\pi^2 g H_0 / L^2 - \bar{C}_a u_{a0} / L + \alpha_1 \alpha_2 + f^2)\mu$$

$$+ \alpha_2(\pi^2 g H_0 / L^2 - \bar{C}_a u_{a0} / L) - f \bar{C}_a v_{a0} / L = 0. \quad (43)$$

#### Stability and oscillation criteria

In the present research the bulk ice viscosity  $\xi$  is taken to be twice the shear ice viscosity  $\eta$ . The standard values of the parameters are given in Table 2. We solve the characteristic equation (43) with different values of  $H_0$ ,  $\eta$ , and  $DT_0$ .  $H_0$  varies from 0.5 to 10.5 m,  $\eta$  from 0 to  $2 \times 10^9$  kg s<sup>-1</sup>, and  $DT_0$  from 1 °C to 21 °C. The instability criterion of the MIZ migration in the ice/air-coupled model can be written by

$$\text{Re}(\bar{\mu}) \begin{cases} < 0 \text{ decaying} \\ = 0 \text{ neutral} \\ > 0 \text{ growing} \end{cases} \quad \bar{\mu} = \mu_1, \mu_2, \mu_3 \quad (44)$$

Table 2. The standard model parameters.

$L = 200 \text{ km}$	$\nu = 5 \text{ m}^2 \text{ s}^{-1}$	$\nu_h = 10^3 \text{ m}^2 \text{ s}^{-1}$
$\Omega = 0.729 \times 10^{-4} \text{ s}^{-1}$	$g = 9.81 \text{ m}^2 \text{ s}^{-2}$	$K = 0.9$
$N = 10^{-2} \text{ s}^{-1}$	$\theta_{B0} = 270^\circ \text{K}$	$C'_w = 0.55 \text{ kg/m}^2 \text{ s}^{-1}$
$C'_a = 1.2 \times 10^3$	$\rho_a = 1.29 \text{ kg m}^3$	$\rho_w = 10^3 \text{ kg m}^3$
$\rho_i = 910 \text{ kg m}^3$	$U_i < 1 \text{ m s}^{-1}$	$\lambda = (65^\circ, 70^\circ, 75^\circ)$

where  $\bar{\mu}$  is the root of the cubic equation (43). The oscillation criterion of the MIZ migration is given by

$$\text{Im}(\bar{\mu}) \begin{cases} = 0 & \text{nonoscillatory} \\ \neq 0 & \text{oscillatory} \end{cases} \quad \bar{\mu} = \mu_1, \mu_2, \mu_3 \quad (45)$$

#### Dependence of eigenvalues on parameters

We compute all the roots of Equation 43. Here  $\mu_1$  is real in the entire space, and  $\mu_2$  and  $\mu_3$  are real

for some values of  $(H_0, \eta, DT_0)$  and are complex conjugates for others.

Figure 7 shows the surface of  $\mu_1 = 0$  in three-dimensional space  $(H_0, \eta, DT_0)$ . This surface divides the space into two parts corresponding to growing and decaying modes. The growing mode appears when  $DT_0$  exceeds some critical value around  $2^\circ \text{C}$ , and is located in the small mean ice-thickness  $H_0$  region. Ice motion corresponding to the eigenvalue  $\mu_1$  is nonoscillatory.

Figure 8 indicates the surface of  $\text{Re}(\mu_2) = 0$  [or

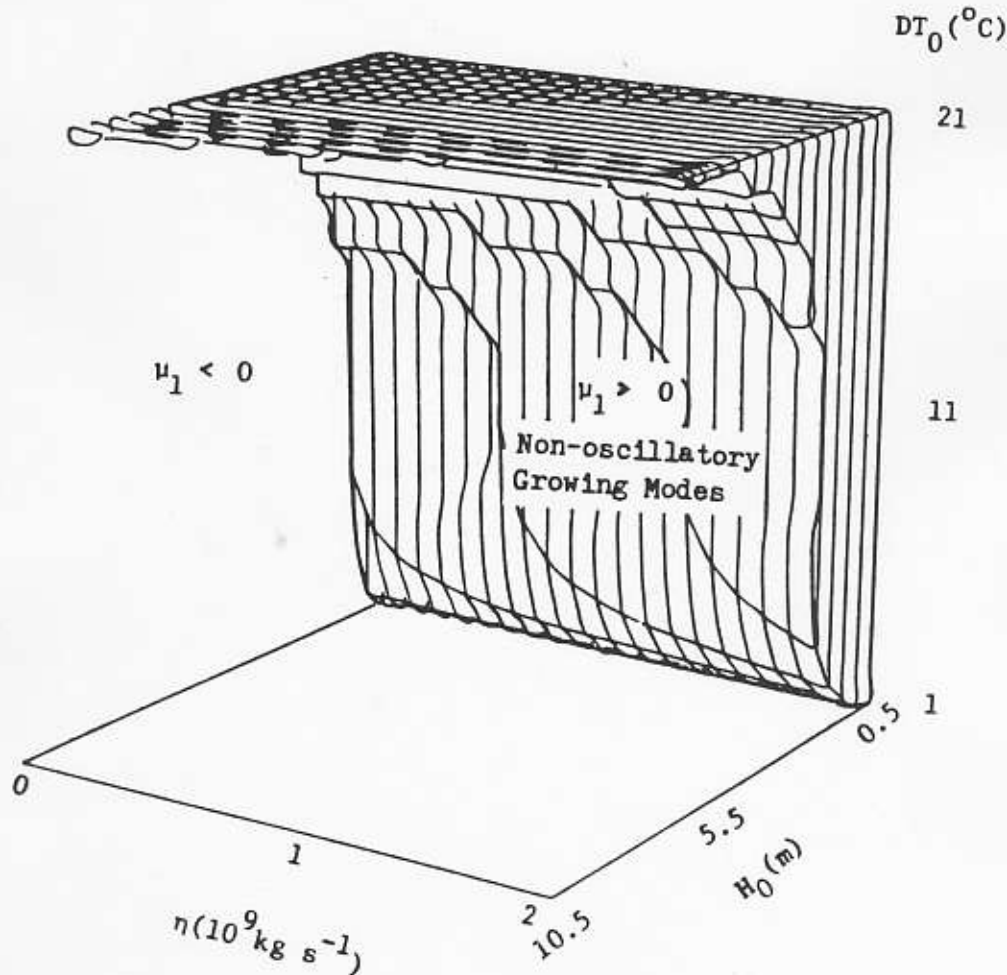


Fig. 7. Separation of nonoscillatory decaying and growing modes referring to real eigenvalue  $\mu_1$ .



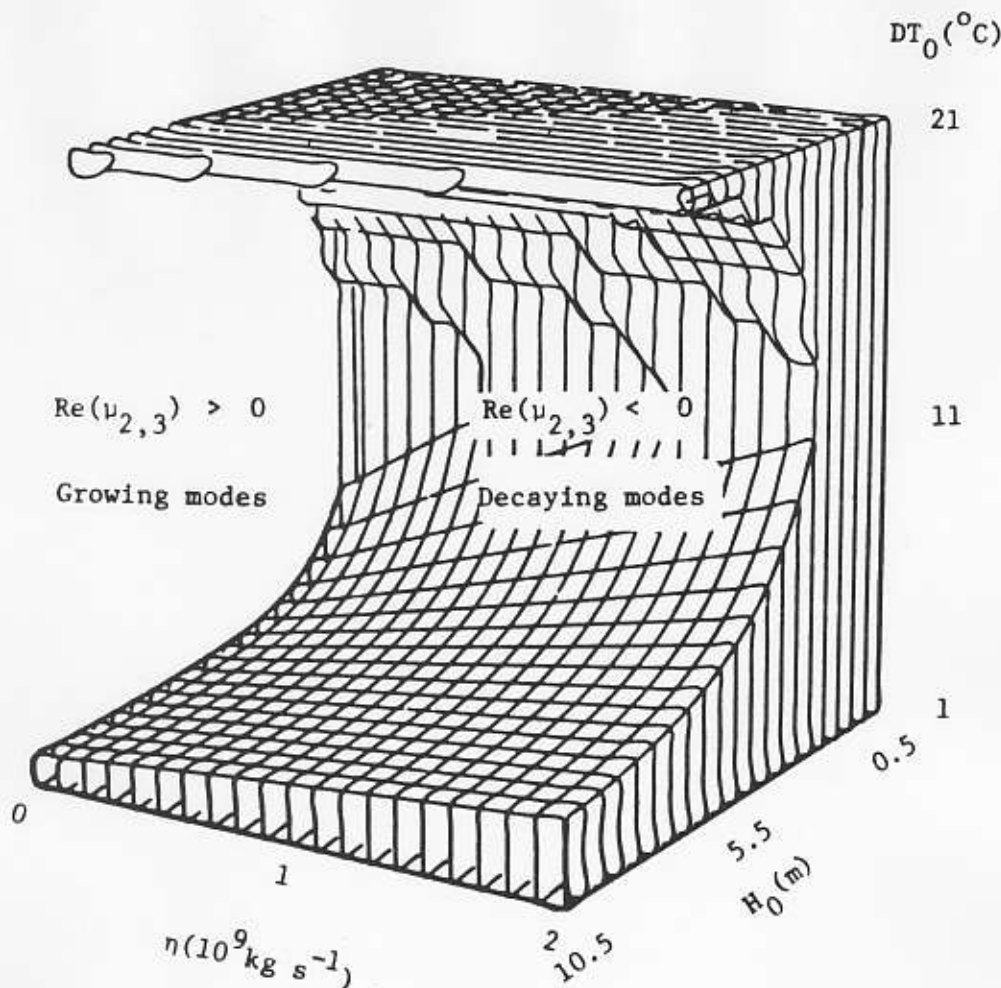


Fig. 8. Separation of decaying and growing modes referring to eigenvalues  $\mu_2$  and  $\mu_3$ .

$\text{Re}(\mu_1) = 0$ ] in the parameter space  $(H_0, \eta, DT_0)$ . This surface separates the whole space into growing and decaying parts. The growing mode is present when  $DT_0$  exceeds some critical value that is a function of  $H_0$  and  $\eta$  (in case of  $H_0 = 4.5$  m and  $\eta = 10^9$  kg s $^{-1}$ ,  $DT_0 = 6^\circ\text{C}$ ), and is concentrated in the large mean ice-thickness  $H_0$  region.

Figure 9 reveals the segregation of nonoscillatory and oscillatory modes relating to  $\mu_2$  and  $\mu_3$ . Comparing Figure 9 with Figure 8, we find that the decaying mode of  $\mu_2$  and  $\mu_3$  is nonoscillatory, whereas the growing mode of  $\mu_2$  and  $\mu_3$  is oscillatory.

Whether ice motion grows or decays substantially depends on the parameter  $DT_0$ . When  $DT_0$  is small, the motive force is so small that it cannot overcome the dissipative effect and does not make ice motion unstable. If  $DT_0$  becomes large enough to overcome the dissipative forces originated by ice stress and friction, ice motion becomes larger.

Whether ice motion is oscillatory or nonoscillatory largely depends on parameter  $H_0$ . In cases of small  $H_0$ , the restoring force is so small that it cannot balance the motive force and cannot produce oscillations. However, when parameter  $H_0$  exceeds some critical value (which varies with  $DT_0$  and  $\eta$ ) and becomes large enough to balance the motive force brought on by the ice breeze generated by surface temperature difference, the ice motion becomes oscillatory.

For the nonoscillatory growing mode, the time during which ice doubles its speed is computed by

$$T_1 = \ln 2 / \mu_1 \quad (\mu_1 > 0). \quad (46)$$

If  $H_0$ ,  $\eta$ , and  $\xi$  are assigned as commonly used values:

$$H_0 = 1.5 \text{ m}, \quad \eta = 10^9 \text{ kg s}^{-1}, \quad \xi = 2 \times 10^9 \text{ kg s}^{-1},$$

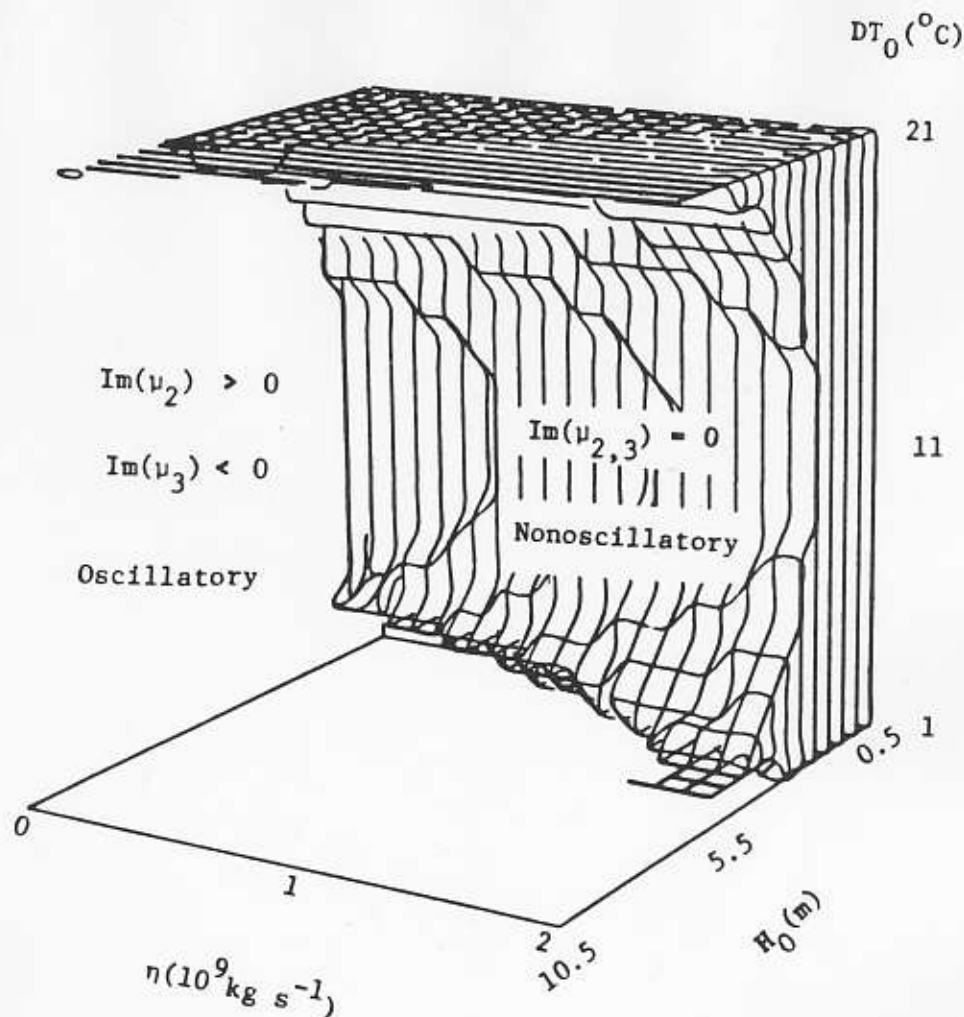


Fig. 9. Separation of nonoscillatory and oscillatory states referring to eigenvalues  $\mu_2$  and  $\mu_3$ .

The doubling time treated as a function of  $DT_0$  is shown in Figure 10, which displays a decrease of  $T_1$  with an increase of  $DT_0$ .  $T_1$  changes from 7.3 hr to 0.73 hr as  $DT_0$  varies from 6°C to 16°C. It takes only 12.4 hr for the ice velocity to increase from 6 cm s<sup>-1</sup>, which is a steady-state estimation, to 20 cm s<sup>-1</sup> when  $DT_0 = 6^\circ\text{C}$ .

The oscillatory growing mode is ice motion due to  $\mu_2$  (or  $\mu_3$ ), whose real part is positive.

$$\text{Re}(\mu_2) = \text{Re}(\mu_3) > 0,$$

the growth rate  $\text{Re}(\mu_{2,3})$  increases with an increase of  $DT_0$  (increase of forcing term), with an enlargement of  $H_0$  (increase of restoring force), and with a reduction of  $\eta$  (decrease of dissipation). With  $DT_0 = 6^\circ\text{C}$ ,  $\eta = 10^9 \text{ kg s}^{-1}$ , and  $H_0 = 4.5 \text{ m}$ , the growth rate is  $0.22 \times 10^{-4} \text{ s}^{-1}$  and the corresponding doubling time is 8.75 hr. The period of growing oscillatory mode, defined by

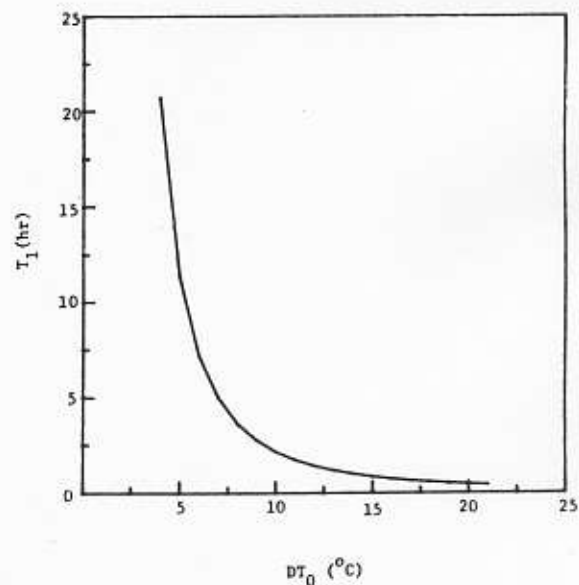


Fig. 10. Dependency of doubling time on  $DT_0$  for the nonoscillatory growing mode.

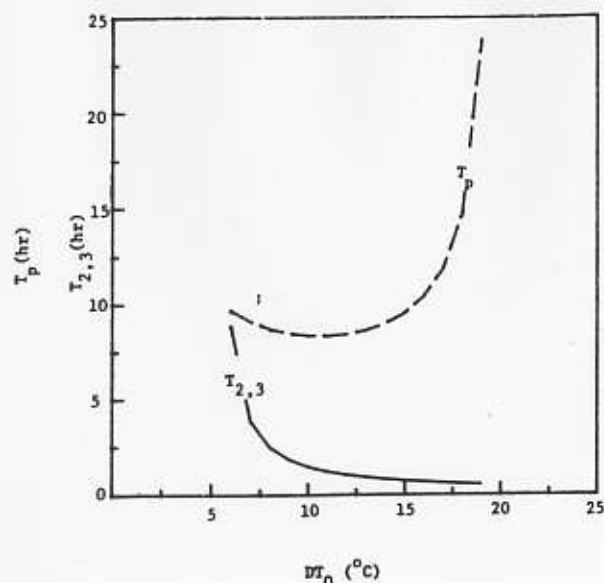


Fig. 11. Dependency of doubling time and period on  $DT_0$  for oscillatory growing mode.

$$T_p = 2\pi / |\text{Im}(\mu_{2,3})| \quad (47)$$

is 9.7 hr.

Figure 11 shows the growth rate and period as functions of  $DT_0$  for  $H_0 = 4.5$  m and  $\eta = 10^9$  kg s<sup>-1</sup>. The growth rate decreases monotonically with an increase of  $DT_0$ . However, the period decreases slightly when  $DT_0$  varies from 6°C to 10°C, and then increases with  $DT_0$  rapidly. When  $DT_0 = 19^\circ\text{C}$ , the period is nearly one day.

### Conclusion

This ice/air-coupled model is intended only to depict the mesoscale processes of ice/air interaction in the MIZ. The synoptic scale pressure gradient associated with the semi-permanent Icelandic low may in addition produce equatorward winds in the Greenland Sea, and large-scale ocean currents near the East Greenland Sea or the Antarctic Peninsula may drive ice drift. These processes are, however, beyond the scope of the current research. Nevertheless, when the ice to open water temperature gradient is strong, the mesoscale feedback mechanism discussed here may become as strong, or stronger, than the synoptic scale and oceanic forcings.

The equatorward and waterward surface winds near the MIZ in the Greenland Sea and the east Antarctic Peninsula are thermally generated by differential surface heating over ice and water. The surface wind has its maximum speed along the

ice edge. The component of surface wind parallel to the ice edge can reach 7 m s<sup>-1</sup> when the temperature gradient in the cross-ice direction is chosen as 0.03 °K km<sup>-1</sup>. This agrees with the observational results at the east Antarctic Peninsula reported by Schwerdtfeger (1979).

The migration of the MIZ is forced by local winds generated by differential surface heating over ice and water near the ice edge. The ice velocity  $V_i$  has sinusoidal forms (38), showing an ice edge jet. The ice velocity has its maximum at the ice edge and decreases iceward. This implies that the ice edge jet is produced indirectly by the thermal effect of the ice/water contrasts.

The ice motion has two bifurcations. It first bifurcates into decaying and growing modes, depending on the mean surface temperature difference over ice and water,  $DT_0$ . When  $DT_0$  is small, the decaying mode predominates. However, when  $DT_0$  exceeds some critical value (i.e., 6°C as  $H_0 = 4.5$  m and  $\eta = 10^9$  kg s<sup>-1</sup>, and 2°C as  $H_0 = 1.5$  m and  $\eta = 10^9$  kg s<sup>-1</sup>), the growing mode appears. The growing mode further bifurcates into nonoscillatory and oscillatory states, depending on ice properties. In cases of small ice thickness, which causes an inadequate restoring force and generally occurs in summer, the ice motion is nonoscillatory. In cases of large ice thickness, which generates an adequate restoring force and is usually present in winter, the ice motion is oscillatory.

This research provides a possible way to predict the MIZ migration. The procedure is as follows:

- Observe surface temperature difference  $DT_0$  and determine the ice properties, i.e.,  $H_0$  and  $\eta$ .
- Use Figures 8 and 9 to determine the pattern of motion, i.e., growing or decaying, oscillatory or nonoscillatory, etc.
- Employ Figures 10 and 11 to determine the growth rate and period (in case of the oscillatory state).
- Observe initial ice velocity.
- Make a prediction of MIZ migration according to the initial ice velocity, growth rate, and period (for the oscillatory state).

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